Investigation of VTOL Landing Control Laws for Low-Speed Flight

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Control laws for the automatic landing of a VTOL aircraft are described. Only the low-speed and touchdown regions of the flight envelope are considered; i.e., the aircraft forward velocity does not exceed 50 fps. The vehicle used for the investigation is the Short SC1 jet-lift VTOL aircraft, and a suitable flight path for this is outlined. A mathematical model of the aircraft is constructed in five degrees of freedom; yawing motion is neglected, as heading is assumed to be held constant for the landing maneuver. Both the model and the flight path are simulated on an analog computer. The landing control laws are designed using conventional principles and operate in the tracking mode. Reference trajectories are set up and control laws described for glide-path holding, track holding, and the speed-range characteristic. As an aid to control law design, a root locus study is carried out in addition to the analog simulation. Results are presented to indicate the performance of the aircraft and its control laws in still air and under the influence of steady and gusting winds. The system described is mechanized readily and provides reasonable performance.

Nomenclature

a = inverse of engine response time constant, sec^{-1}
F_t = thrust excess, lb
H^* = altitude error, ft
H_t = hover thrust, lb
H_{ref} = reference altitude, ft
I_{xx}, I_{yy} = moments of inertia about rolling and pitching body datum axes, slug-ft^2
$K_a, K_1, \ldots$ = constants
P, Q = rates of roll and pitch about body datum axes, deg/sec
s = Laplace variable
S_a, S_p = aileron and pitch stick deflections, in.
$\dot{S}_x, \dot{S}_y, \dot{S}_z$ = velocity components along, across, and normal to the runway (S_x refers to the longitudinal range from the touchdown point), fps
S_x^* = error between reference and aircraft longitudinal distances, ft
$\dot{S}_{xref}$ = reference ground speed, fps
S_{yref} = reference lateral displacement, ft
t = time, sec
T_1, etc. = time constants, sec^{-1}
T_d = demanded lift engine thrust, lb
T_e = lift engine thrust, lb
T_h = point of discontinuity in lift engine thrust characteristic, lb
T_i = initial lift engine thrust, lb
T_x, T_y = rolling and pitching moments due to engine and reaction control forces, lb-ft
U, V, W = velocity components along the forward, lateral, and normal body datum axes, fps
U_r, V_r, W_r = relative wind velocities along the forward, lateral, and normal body datum axes, fps
w_x, w_y, w_z = wind components along, across, and normal to the runway, fps
α = glide path deviation angle, rad
β = angle between forward body datum axis and the direction of lift engine thrust (thrust vector angle), deg
β_d = demanded thrust vector angle, deg
β_i = initial thrust vector angle, deg
λ_d = lift engine throttle angle (normalized about hover thrust level), deg
σ = track deviation angle, rad
θ, ϕ = attitude angles in pitch and roll, deg
θ_d, ϕ_d = demanded pitch and roll angles, deg

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I. Introduction

THE object of this paper is to define the problem considerations associated with the control laws for a VTOL aircraft automatic landing system and to detail the necessary system modeling techniques. The design of such a system is a complex problem. The rationale for selecting the control laws is to provide optimal performance on a predetermined flight path based on a minimum fuel consumption trajectory. The pilot workload associated with such a task would be great without a landing control aid. The pilot needs to control the aircraft in order to aim at the hover point while the speed progressively reduces, and he needs to be able to initiate and control the deceleration toive at the hover point with near zero speed. In addition, it ihantageous that VTOL aircraft operations can be carried out in poor-visibility conditions.

The paper deals with the automatic landing of the VTOL aircraft in the low-speed and touchdown regions of the flight envelope. Low-speed flight is defined as the condition between hover and a forward velocity of 50 fps. Touchdown is that region between the hover point and the touchdown point. The model used is based on the Short SC1 jet-lift VTOL aircraft.[1] This is a small delta wing machine having one propulsion engine and four lift engines. The latter can be rotated about a transverse axis for acceleration and deceleration purposes.

Although it is difficult to generalize about the landing approach paths to be used by VTOL aircraft because no clearly defined operational characteristics are available, a flight path that might be suitable for the SC1 is suggested here. This is based on Potter's work[2] and covers the complete approach region, as shown in Fig. 1. However, the aircraft model used is valid only for speeds up to about 50 fps, and so just a part of the total trajectory is needed. Data for this region also are taken from Ref. 2 and are summarized in Fig. 1. To represent

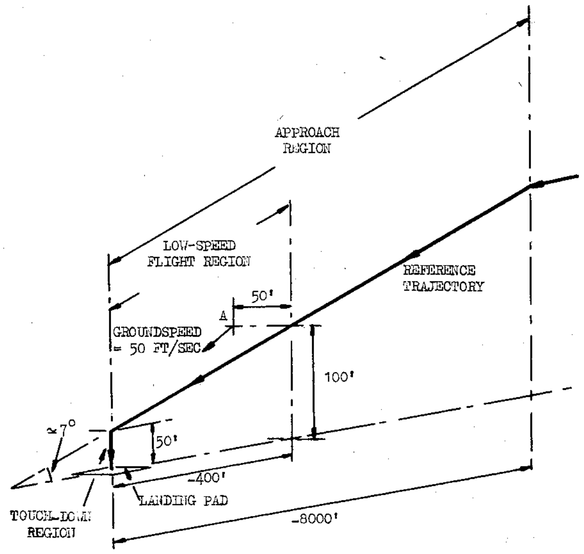

Fig. 1 Suitable landing path for a VTOL aircraft such as the SC1 (not to scale). Point *A* represents the initial condition of the aircraft for tests.

a severe but fair test to the control system, a lateral displacement from the reference track is included.

Control of the aircraft in the low-speed flight and touchdown regions must take place in three dimensions: glide-path holding, track holding, and the speed-range relationship. These correspond to the vertical, lateral, and longitudinal translational motions, respectively. For a glide-path holding, lift engine thrust is used, and track holding is by means of bank angle control with heading fixed. In this way, yaw can be neglected without making the simulation particularly unrealistic. Aircraft control in pitch and roll for low-speed flight is by means of reaction controls. In the case of speed-range control, two possibilities exist: thrust vector angle or pitch attitude angle. Although it is not unreasonable to use pitch attitude angle as a fine control around the hover point, it is preferable to use engine tilt in the approach region because of the nonlinear relationship between pitch angle and the speed-range characteristic.[1] It is, therefore, also more convenient to continue using engine tilt for the low-speed flight region. In addition, it is convenient to hold propulsion engine thrust constant at idle for the low-speed flight maneuver.

Although limited previous work has been published which is directed specifically toward the VTOL automatic landing problem, a considerable effort has been expended in designing automatic landing systems for conventional aircraft. Although the problems involved in the VTOL case are more complex, there are obvious similities between the two. Blakelock,[3] Musker and Henman,[4] Merriam,[5] and Dyer[6] are some who describe rous approaches to the conventional aircraft automatic landing problem.

With regard to the design of VTOL control systems, Short Brothers did some work on an automatic landing system for the SC1 aircraft in the early 1960's. Although this never was completed, many of the principles used are incorporated in the control laws described here. More recently, optimal control theory has been applied to VTOL control systems, in the design of both autostabilization systems and automatic landing systems. Although the automatic landing system is of primary concern here, the autostabilizer forms an integral part of the complete system. The autostabilizers used in the simulation were developed for the SC1 aircraft by Short Brothers and are conventionally designed systems. However, Dyer,[6] Murphy and Narendra,[7] Nakagawa et al.,[8] and Rynaski[9] all describe the application of optimal control methods to the design of stabilization systems.

It is only during the past eight years or so that literature has been published regarding VTOL automatic landing systems. As part of the NASA-ERC V/STOL Avionics Program in the United States, computer recommendations for an automatic approach and landing system for V/STOL aircraft have been made.[10] In addition, an independent control system synthesis was performed using linear optimal control techniques, and Hoffman et al.[11] subsequently have published work on this system. Various other papers relating to helicopter guidance and control are given in Ref. 14.

II. Aircraft Model

A. Introduction

VTOL aircraft simulation poses problems not met with in the simulation of conventional aircraft. During wing-borne flight, the analysis is like that of a conventional aircraft, but, as the transition region is entered, the aerodynamic forces acting on the vehicle become highly nonlinear. Also, the effect of the (nonlinear) lift engine characteristic must be taken into account. In the low-speed flight region considered here, the aircraft has very little inherent stability, and its control is dependent on the characteristics of the autostabilizers and the lift engine.

The main factor affecting the form of the equations of motion in the low-speed flight region is the absence of a clearly defined direction in which the resultant velocity vector will lie. The velocity vector in a conventional aircraft is confined in direction to a small region about the aircraft's forward body axis, but a VTOL aircraft in low-speed flight may move with almost equal ease in any direction. Flight-path-based axis systems are therefore unsuitable for VTOL use, and in the model developed here the equations of motion are based on an axis system fixed in the aircraft.

The model described in this section is derived from the simulation by Perry and Chinn,[12] although it is developed for landing system work rather than for a study of handling qualities. Bearing this in mind, the following modifications have been incorporated: 1) attitude autostabilizers as used on SC1, 2) a more realistic lift engine characteristic, and 3) a thrust vector angle simulation. A lift loss term also was incorporated, although it was found to have no significant effect on landing control law performance.

B. Aerodynamics: Equations of Motion for Low-Speed Flight of a VTOL Aircraft

There are many textbooks available which derive the generalized equations of motion for a symmetrical, rigid aircraft (e.g., Blakelock,[3] and Etkin[13]). Using the generalized equations as a starting point, Perry and Chinn make developments for the VTOL low-speed flight case. To do this, the forces and moments acting on the aircraft are analyzed and then substituted into the generalized equations.

The analysis described in Ref. 12 differs from the well-known small perturbation theory for conventional aircraft, as several terms that usually are neglected now need to be retained. Because the lateral velocity components can be relatively large in low-speed VTOL flight, interaction terms in the equations of motion cannot be ignored. Also, several approximations are made in evolving the equations in order to reduce the amount of computational equipment required. Most of the approximations are concerned with resolving the velocity components computed with respect to aircraft axes into velocity components over the ground and take the form of small angle approximations.

As the work described here is concerned only with flight at speeds below 50 fps, the aerodynamic forces acting on a jet-borne aircraft can be considered relatively small. Perry and Chinn assume that the aerodynamic effects can be represented adequately by forces and moments about each axis which are simply proportional to the velocity component along that axis. Numerical values for these aerodynamic effects are taken from Ref. 12. A listing of the aerodynamic equations is given in the Appendix.

C. Autostabilizers

As the inherent damping of the SC1 aircraft is low, some form of artificial stabilization is necessary to insure reasonable control. The autostabilizers described below are for stabilization in the pitch and roll axes. Because heading is taken as constant, a perfect yaw autostabilizer is assumed.

The autostabilizers outlined here were designed by Short Brothers and intended (although not used) for automatic control of the SC1. To make this possible, they were designed to provide an accurate response and to include a true attitude mode. In practice, they have a gain change with speed and can operate on reaction and aerodynamic controls. Vertical gyros provide the attitude reference, and an integral of error signal gives steady-state accuracy. The forms of the control laws in pitch and roll are as follows

Pitch

$$[K_1/(1+T_1s)]\ (\dot{T}_y/I_{yy}) = K_2S_p - [K_3/(1+T_2s)]\ddot{\theta} - K_4\dot{\theta} - K_5(\theta-\theta_d) \qquad (1)$$

Roll

$$[K_6/(1+T_3s)]\,(\dot{T}_x/I_{xx}) = K_7S_a - [K_8/(1+T_4s)]\ddot{\phi} - K_9\dot{\phi} - K_{10}(\phi-\phi_d) \qquad (2)$$

D. Lift Engine Characteristic

The lift engine characteristic can be represented by a first-order lag with a time constant of 0.11 sec, together with a limitation on the maximum rate of change of thrust of 3500 lb/sec.[12] In the SC1, there are four lift thrust engines, and their maximum and minimum combined thrusts are 8400 and 2400 lb, respectively. The relationship between the thrust demanded (T_d,lb) and the thrust delivered (T_e, lb) is therefore nonlinear, but it can be divided into two linear parts with a discontinuity at time t_1. Figure 2 shows the lift engine response to a step thrust demand from minimum to maximum. In addition, the demanded thrust is dependent on the position of the lift engine throttle. If the throttle angle (λ_d, deg) is normalized about the approximate hover thrust level (H_t, lb), it is related to the demanded thrust by

$$T_d = (K_s)\lambda_d + H_t \qquad (3)$$

where K_s is the demanded thrust (lb) per degree of throttle angle.

E. Thrust Vector Angle System

The lift engines of the SC1 aircraft are arranged in a group about the aircraft's center of gravity. They may be tilted about a transverse axis, and the direction in which their thrust acts (β) is known as the thrust vector angle. It is defined as the angle between the centerline of the engines and the forward body datum axis of the aircraft. The range of β is from 67° in acceleration to 102° in retardation, and the rate of change of β is fixed at 5 deg/sec (K_t). If the initial angle is β_i deg, then for a step demand to β_d at time $t=0$ the ideal response characteristic is

$$\beta(t) = \beta_i \pm (K_t)t \text{ for } t \leq t_a \qquad (4a)$$

(positive for increasing angles, negative for decreasing angles), and

$$\beta = \beta_d \quad \text{for } t > t_a \qquad (4b)$$

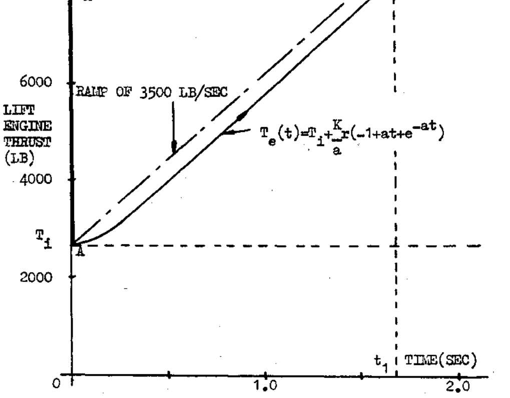

Fig. 2 Lift engine response for a step thrust demand from minimum to maximum.

F. Simulation

The generalized equations of motion of the aircraft and its associated systems take the form of ordinary differential equations and algebraic equations, and these may be solved conveniently using an analog computer.

III. Automatic Landing Control Laws

A. Introduction

To obtain suitable control laws, the aircraft model, the simulated flight path, and the simulated guidance equipment were set up on an analog computer. Control law configurations and parameters then were adjusted by trial-and-error methods in conjunction with theoretical analyses until satisfactory dynamic and steady-state responses resulted. The conventional design techniques employed dictate that interactions among the longitudinal, lateral, and vertical modes are neglected. This is reasonable, provided that coupling among the modes is not high.

B. Reference Trajectories

Reference Trajectories for the Low-Speed Flight Maneuver

For the control of vertical motion in the low-speed flight phase, a reference altitude trajectory is generated using longitudinal range as the independent variable. If H_{ref} is the reference altitude (ft) taken as positive upwards, then the equation of the reference trajectory for the flight path described in Sec. I is

$$H_{ref} = -(0.125)S_x + 50.0 \qquad (5)$$

where S_x is the longitudinal range from the touchdown point (ft). Note that S_x is, by convention, negative.

For the control of longitudinal motion, ground speed is used as the reference variable; ground speed itself is a function of longitudinal range. For this exercise, reference ground speed ($\dot{S}_{xref}$) is made proportional to S_x in the following manner

$$\dot{S}_{xref} = -(0.125)S_x \qquad (6)$$

This law is a convenient means of satisfying the conditions given in Fig. 1. It is not intended as a speed-range law for the entire approach region.

As a result of Eq. (6), the relationship between ground speed and time is exponential. For an efficient practical landing system, this may not be ideal because a large amount of fuel would be used while proceeding at a low forward velocity. To overcome this problem, an inverse parabolic or exponential reference ground speed/range law, or the nonlinear relationship resulting from Potter's work,[2] could be used.

For the control of lateral motion, the reference track is taken to be a predetermined straight line in space terminating at the hover point, i.e.

$$S_{yref} = 0 \qquad (7)$$

Touchdown Region Reference Trajectory

When conventional aircraft perform the "flare-out" maneuver, a good landing has been found to result if altitude rate is made proportional to altitude.[5] In many ways, the touchdown control of a VTOL aircraft can be likened to the flare-out maneuver of a conventional aircraft, the difference being, of course, that the former has no forward velocity relative to the ground. The reference law used here to transfer the aircraft from the hover point to the touchdown point is

$$\dot{H}_{ref} = (0.10)S_z \qquad (8)$$

where $\dot{H}_{ref}$ is the reference altitude rate (fps) taken as positive upwards. It should be noted that S_z is, by convention, positive downwards as it is derived from the aircraft equations of motion.

C. Control of Low-Speed Flight

Vertical Motion

Control Law. If the aircraft's altitude at any instant is $-S_z$ (ft), then the altitude error from the reference path [Eq. (5)] is given by

$$H^* = -S_z - H_{ref} \qquad (9)$$

If a beam-oriented system is used for guidance in a manner similar to conventional ILS information, then altitude would not be measured directly, but a measure of glide-path deviation angle (α) would be obtained. As both the glide-slope angle and the glide-path deviation angle are small, α may be approximated by the following equation:

$$\alpha(\text{rad}) \approx H^*/(-S_x + 400.0) \qquad (10)$$

where α is taken as positive when the aircraft is above the reference flight path.

In addition to the measurement for α, an accelerometer can be used to determine the aircraft's accleration normal to the reference flight path. For the work described here, the vertical acceleration component is considered to be $\dot{W}$, but for greater accuracy corrections could be made with respect to incidence, pitch rate, forward acceleration, etc., to obtain acceleration normal to the reference path.

The control input for vertical maneuvers is the angle of the lift engine throttle lever, and, in order to form a suitable control law, this must be related to the available measurements. A satisfactory relationship was determined to be (from the simulation)

$$\dot{\lambda}_d = [1/(1+T_5 s)]\{(K_{11})\dot{W} - [K_{12}/(1+T_6 s)]\dot{\alpha} - (K_{13})\alpha\} \qquad (11)$$

where $\dot{\lambda}_d$ is the demanded rate of change of lift throttle angle.

It can be seen from Eq. (10) that, for a fixed value of H^*, α depends on the longitudinal range S_x. In order to keep the overall system gain constant, K_{12} and K_{13} of Eq. (11) themselves must vary with range.

Root Locus Analysis. In order to analyze the control law further, a root locus study is carried out using a linearized version of the aircraft model. A block diagram for the arrangement is shown in Fig. 3. Incorporated in the inner loop are a pure integration (arising from the servo characteristic) and a simple lag with a time constant of 0.1 sec corresponding to the smoothing network. The effect of the integration is to cancel the zero at the origin which arises from the aerodynamics. This increases the "type" of the overall system from type 1 to type 2.

The root locus plot for the complete system is shown in Fig. 4. Typical dominant closed-loop poles are indicated. The position of these verifies that the system is stable, with a

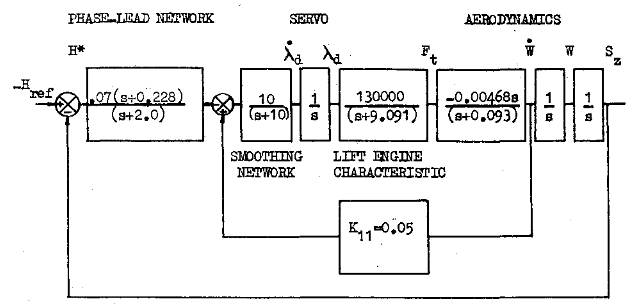

Fig. 3 Simplified aircraft model and vertical motion control law normalized about the hover point.

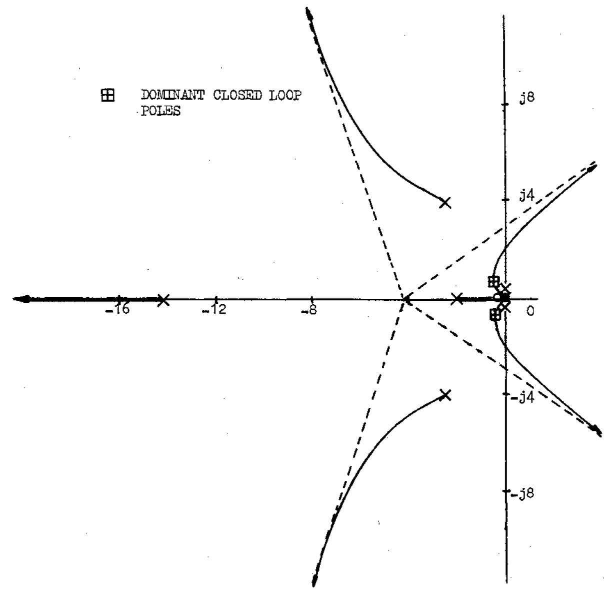

Fig. 4 Root locus plot for the vertical motion system.

damping ratio of 0.53 and a natural frequency of 0.8 rad/sec. The phase-lead network incorporated in the outer loop is the practical means of obtaining derivative control. This network stabilizes the system because the zero at $s = -0.228$ effectively pulls the locus emerging from the two poles at the origin into the left-half s plane. The pole of the network is selected at a value around $s = -2.0$, so that noise on the signal is not overemphasized.

Longitudinal Motion

Longitudinal movements of the aircraft are implemented by thrust vector angle control. This produces acceleration changes in the longitudinal plane, and the object of the control law is to bring the aircraft to a halt at the hover point. Having developed a law defining reference ground speed [Eq. (6)], the following relationships are introduced

$$S_{x_{ref}} = -\int \dot{S}_{x_{ref}} \mathrm{d}t \qquad (12)$$

$$S_x^* = S_{x_{ref}} - S_x \qquad (13)$$

where $S_{x_{ref}}$ is the reference longitudinal distance from the touchdown point, and S_x^* is the error between the reference and aircraft longitudinal distances. A suitable control law relating demanded thrust vector angle to S_x^* was determined to be of the form

$$(\beta_d - 90°) = [1/(1+T_7)]\{(K_{14})\dot{S}_x^* + (K_{15})S_x^*\} \qquad (14)$$

Pitch angle is assumed to remain at constant through autostabilization.

The equations governing longitudinal motion are inherently nonlinear due to the thrust vector angle system. Also, the cosine of the thrust vector angle (cos β) has to be multiplied by the lift engine thrust (T_e) to generate longitudinal motion. The most straightforward means of analysis is by analog computer simulation, and this shows that Eq. (14) gives reasonable dynamic performance. Also, it shows that the system acts rather like a type 1 control system such that a steady-state position error does not arise for the landing; i.e., the aircraft comes to rest over the landing pad.

Lateral Motion

As heading is assumed fixed for lateral maneuvers, roll control is required to keep the aircraft on the predetermined track in space. This is a straight line terminating at the hover point, and displacement measurements from this line are angular. These are similar to the measurements for glide-path deviation angle; i.e., for a fixed displacement from the track, the deviation angle (σ) is dependent on the longitudinal range. Hence

$$\sigma(\text{rad}) \approx S_y / (-S_x + 400.0) \qquad (15)$$

where S_y is the lateral displacement from the predetermined track (ft.). A three-term (proportional-integral-derivative) law is proposed for lateral control

$$\phi_d = -[1/(1+T_8 s)]\{[K_{16}/(1+T_9 s)]$$

$$\dot{\sigma} + (K_{17})\sigma + (K_{18})(\sigma/s)\} \qquad (16)$$

Because the track deviation angle is range dependent, K_{16},K_{17} and K_{18} must vary with range to keep the overall system gain constant.

A block diagram illustrating the lateral control law used on a simplified version of the aircraft model is given in Fig. 5, and analysis shows this system to be type 2. Although the integral term increases the type of the system, it decreases the speed of response.

Wind Effects

In order to analyze the behavior of lateral aircraft motion with respect to a steady crosswind, an extra input to the system (w_y) is included. The position and nature of the additional input are shown in Fig. 5. By letting w_y be a step function and using the final value theorem of Laplace, the steady-state position error from the reference track can be calculated. For the control law used, zero steady-state position error results.

A similar technique can be used for determining the steady-state error resulting from vertical wind disturbances. Although no steady vertical wind component is assumed to act on the aircraft, analysis shows that the presence of such a component would result in no steady-state position error. The steady-state position error evolving from a constant velocity longitudinal wind disturbance can be obtained from computer simulation and is in some ways similar to the lateral case if no integral term were to appear in the control law; i.e., a finite steady-state position error would be expected to exist. However, the results given in Sec. III E indicate no position error. The reason for this discrepancy arises from the manner in which the reference signal is obtained in the longitidunal case. It can be seen from Eq. (12) that $S_{x\text{ref}}$ is obtained from the integration of $\dot{S}_{x\text{ref}}$, which is, in turn, directly proportional to longitudinal distance S_x [Eq. (6)]. Thus, a pure integration term exists in the generation of the reference quantity. From simulation, the presence of this term can be shown to result in zero steady-state position error for the headwind and tailwind disturbance cases. However, it should be emphasized that, if $S_{x\text{ref}}$ were not obtained in this way, steady-state longitudinal position errors could arise.

In order to overcome this problem, additional compensating networks could be incorporated in the longitudinal control equation. To eliminate a steady-state position error

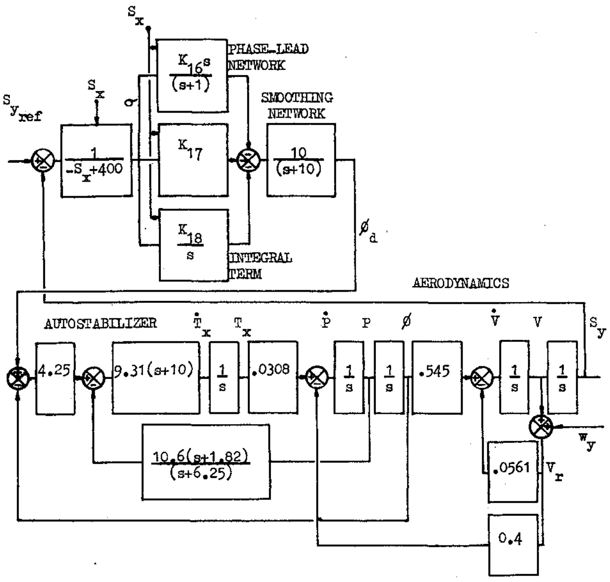

Fig. 5 Block diagram for the control of lateral motion using a simplified version of the aircraft model.

completely, an integral term would be required in a manner similar to that used in the lateral control law. However, care would be needed incorporating such a term because response speed could be affected seriously.

D. Touchdown Control

For vertical flight from the hover point to the touchdown point, the vertical control law described by Eq. (11) can be adapted for altitude and altitude rate inputs rather than glide-path deviation angle and its rate of change. Altitude measurements obtained from an airborne electronic altimeter should provide sufficiently accurate and noise-free data; the use of such an altimeter would seem a prerequisite for the mechanization of a satisfactory touchdown control law. If an accelerometer were employed in conjunction with the altimeter, a control law similar to that described by Eq. (11) could be used.

E. Specimen Results

Test Program

The tests carried out in the low-speed and touchdown phases are aimed at illustrating the performance of the control laws. Although the tests are based on practical conditions, they serve as a general guide to performance rather than as a strictly accurate representation. The initial conditions for the low-speed flight region are summarized in Fig. 1. The touchdown phase commences at the finish of the low-speed region, and its nominal initial conditions in still air are those for stationary hovering flight at 50 ft alt. In addition to testing the control laws from predetermined initial conditions, disturbances are included in the test program. For the aircraft landing case, external disturbances take the form of steady winds and gusts, and the simulation allows both of these to be incorporated.

Results

Figure 6 shows the response of the control laws in low-speed flight for the still air case. Note that the lateral control law gives slow response (Fig. 6c). This is caused by the integral term; if this is removed, faster response results. However, the effects of crosswinds must be taken into account. If an integral term is not included in the control law, a large steady-state position error results when a crosswind is introduced. The law used is free from this (Fig. 7).

The presence of a headwind or tailwind on longitudinal motion of the aircraft does not result in a steady-state position

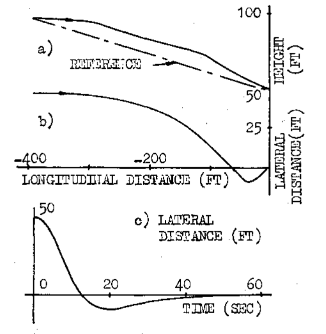

Fig. 6 Responses for low-speed flight in still air.

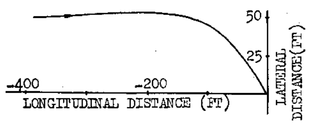

Fig. 7 Response for low-speed flight with a steady crosswind of 25 fps.

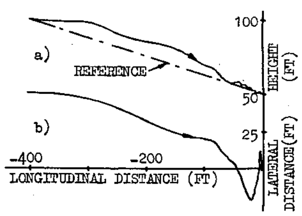

Fig. 8 Responses for low-speed flight with a 50 fps headwind plus gusts.

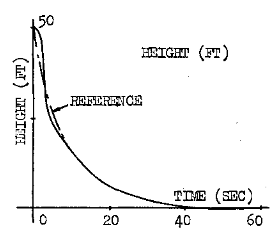

Fig. 9 Time response for the touchdown maneuver in still air.

error. Figure 8a is the height-against-distance trace for a 50 fps headwind together with gusts acting on the aircraft, and the lateral-distance-against-longitidunal distance trace for the same conditions is shown in Fig. 8b. Note that gusts act in all planes. These results show that, although headwind has little effect on performance, gusts disturb the trajectories, especially in the lateral direction. One reason for the lateral mode being more sensitive than the longitudinal mode is that different aerodynamic characteristics exist in the two modes. However, differences between the forms of the longitudinal and lateral control laws probably play a larger part. Tests performed on the touchdown control law are shown in Fig. 9. The reference trajectory is followed well after the initial disturbance, and system performance is satisfactory.

IV. Conclusions

The conventional approach to control system synthesis used here is a well-accepted procedure; measurements that should be obtainable readily from either the guidance system or the aircraft instrumentation have been operated on by compensating networks to produce the control laws. However, it should be noted that there is some conflict between obtaining adequate steady-state performance and satisfactory dynamic response in some results shown in the previous section.

Although the tests on the system represent severe conditions, the performance of the control laws is only fair. In the vertical plane, there is an error between the aircraft altitude and the reference altitude during the maneuver, and the response in the lateral plane is slow. Although constant winds do not give rise to steady-state errors, gusts affect the aircraft's position, particularly in the lateral plane. Other shortcomings are the following:

1) It has been necessary to decouple the longitudinal, lateral, and vertical modes in order to synthesize the control laws. When these are applied to the complete nonlinear model, where cross-coupling between modes takes place, performance is affected.

2) Control laws have been designed only for the tracking mode. The construction of suitable terminal control laws (i.e., those which funnel the aircraft into the hover point along either a straight-line path that is not predetermined or a free path, possibly with constraints on variables) is difficult because of the time-varying nature of the gains which would need to be calculated.

The study considered in this paper is a preliminary exercise to developing a model and control laws for the entire landing region. It is envisaged that the laws described here can be used for such work, provided that correct gain scheduling is performed.

Appendix: Aerodynamic Equations of Motion for the Short SC1 Aircraft in Low-Speed Flight

1) Longitudinal Motion

a) Pitching Motion

$$\dot{Q} = (K_a) T_y + (K_b) U_r \tag{A1}$$

$$Q = \int \dot{Q} dt \tag{A2}$$

$$\dot{\theta} = Q \cos \phi \tag{A3}$$

$$\theta = \int \dot{\theta} dt \tag{A4}$$

b) Longitudinal Translational Motion

$$\dot{U} = (K_c) T_e \cos\beta + (K_d) - (K_e) U_r$$
$$- (K_f) \sin \theta - (K_g) WQ \tag{A5}$$

$$U = \int \dot{U} dt \tag{A6}$$

$$\dot{S}_x = U + W \sin \theta \tag{A7}$$

$$U_r = U + w_x - w_z \sin\theta \tag{A8}$$

$$S_x = \int \dot{S}_x dt \tag{A9}$$

2) Lateral Motion

a) Rolling Motion

$$\dot{P} = (K_h) T_x - (K_i) V_r \tag{A10}$$

$$P = \int \dot{P} dt \tag{A11}$$

$$\dot{\phi} = P \tag{A12}$$

$$\phi = \int \dot{\phi} dt \tag{A13}$$

b) Lateral Translational Motion

$$\dot{V} = -(K_j) V_r + (K_k) \cos\theta \sin\phi + (K_l) WP \tag{A14}$$

$$V = \int \dot{V} dt \tag{A15}$$

$$\dot{S}_y = V - W\sin\phi \tag{A16}$$

$$V_r = V + w_y + w_z\sin\phi \tag{A17}$$

$$S_y = \int \dot{S}_y \mathrm{d}t \tag{A18}$$

3) Vertical Motion

a) Vertical Translational Motion

$$\dot{W} = -(K_m)T_e\sin\beta + (K_n)\cos\theta\cos\phi - (K_0)$$
$$VP + (K_p)UQ - (K_q)W_r \tag{A19}$$

$$W = \int \dot{W}\mathrm{d}t \tag{A20}$$

$$\dot{S}_z = -U\sin\theta + V\sin\phi + W \tag{A21}$$

$$W_r = W + w_x\sin\theta - w_y\sin\phi + w_z \tag{A22}$$

$$S_z = \int \dot{S}_z \mathrm{d}t \tag{A23}$$

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